**Homework 4**

**P15.1.6** (a) Derive the transfer function of the circuit of Figure P15.1.6; (b) determine *L* and *C* so as to have a second-order, lowpass, normalized Butterworth response, with *R*1 = 2 Ω and *R*2 = 1 Ω; (c) scale the parameters so as to have *R2* = 10 kΩ and a 3-dB cutoff frequency *f*0 = 10 kHz.



**Solution:** (a) The impedance of *C* in parallel with *L* and *R*2 is:= . It follows that:  .

(b) Substituting *R*1 = 2 Ω and *R*2 = 1 Ω, the denominator becomes: . For a normalized Butterworth response,  and . This gives  = 1.6730 F and  = 0.8966 H, or = 0.4483 F and  = 3.3460 H, with *ω*0 = 1 rad/s.

(c) To scale, *RL* = *R*2 to 10 kΩ, *km* = 104; to scale *f*0= 10 kHz, *kf* = ; hence ≡  = 5.3254 nF, or 1.4269 nF, and  = 0.1427, or 0.5325 H.

**P15.3.1** Specify the response **VO**/**VI** of the filter circuit of Figure P15.3.1, and determine the passband gain.



**Solution:** As *ω* → ∞, the inductors behave as open circuits, and the source and op amp are isolated, with the noninverting input connected to ground. Hence, **VO** = 0. When *ω* → 0, the inductors behave as short circuits, with the op amp in the noninverting configuration having a gain of (1 + 2/1) = 3. It follows that the filter is a second-order, lowpass filter of passband gain of 3.

**P15.4.4** Design a broadband bandpass filter having 3-dB cutoff frequencies of 100 Hz and 10 kHz and a passband gain of 2 using 0.2 μF capacitors. The filter is to have a very high input impedance.

**Solution:** For the lowpass filter, *Rfl* = 1/(*ωc*2*Cfl*) = 1/(2*π*×10,000×2×10-7) 79.58 ≅ 80 Ω. The gain of -2 can be achieved by making *Rrl* = 40 Ω. For the high-pass filter, *Rrh* = 1/(*ωc*1*Crh*) = 1/(2*π*×100×2×10-7) = 7958 ≅ 8 kΩ = *Rfh*, so that the gain of this stage is -1. To achieve a very high input impedance, a unity-gain amplifier may be added at the input.



**P15.4.7** It is required to design a third-order highpass Butterworth filter using a first-order highpass filter cascaded with a second-order highpass, noninverting filter of the type shown in Figure 15.4.1, reproduced in Figure P15.4.7 using a unity-gain amplifier. The filter should have a gain of 20 dB and a 3-dB cutoff frequency of 10 krad/s, using 0.1 µF capacitors. Determine the required values of resistances.



**Solution:** From Table 15.2.2, the second order filter of a normalized third-order Butterworth filter should have *Q* = 1. From Equation 15.4.2, *R2* = 2 Ω and *R*1 = 0.5 Ω. To move the cutoff frequency from 1 rad/s to 10 krad/s requires a frequency scale factor *kf* = 104. To use capacitors of 0.1 μF requires a magnitude scale factor of *km* = 103. Resistances are multiplied by *km*, so that *R*1 = 103×0.5 = 500 Ω and *R*2 = 2 kΩ.

To have a first-order highpass filter, an active *RC* filter having *Rr* = 1/*ωchC* =

1/(104×10-7) = 1 kΩ and *Rf* = 10 kΩ.